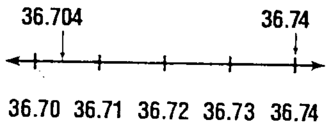
Name ______________________________ Date ______________

Study Guide Worksheet 2-1

Comparing and Ordering Decimals

Which is greater, 36.74 or 36.704?

You can compare the decimals on a number line.
Numbers to the right are greater than numbers to the left.



You can also compare decimals by comparing the digits in each place-value position.

Find the first place in which the digits are different.

36.704
36.74

Compare the digits.

0 is less than 4.
$0 < 4$

The decimal with the greater digit is greater.

$36.704 < 36.74$

Draw a number line to show which decimal is greater.

1. 0.39, 0.35

2. 1.95, 2.02

3. 6.55, 6.50

Complete with $<$, $>$, or $=$ to make a true sentence.

4. 8.05 ____ 8.5 **5.** 0.76 ____ 0.67 **6.** 18.20 ____ 18.2

7. 7.004 ____ 7.044 **8.** 6.79 ____ 6.8 **9.** 29.922 ____ 29.299

Order each set of numbers from least to greatest.

10. 0.067, 0.6, 0.76, 0.07

11. 56.2, 55.6, 52.2, 56.02

12. 600.09, 609.06, 600.9, 609.9

13. 0.88, 0.9, 0.08, 0.89

Glencoe Division, Macmillan/McGraw-Hill

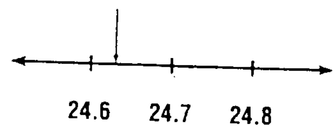
Name ______________________ Date ______________

Study Guide Worksheet 2-2

Rounding Decimals

Round 24.625 to the nearest tenth.

You can use a number line.



Find the approximate location of 24.625 on the number line.

24.625 is closer to 24.6 than to 24.7.
24.625 rounded to the nearest tenth is 24.6.

You can also round without a number line.

Find the place to which you want to round.

24.625

Look at the digit to the right.
If the digit is less than 5, round down.
If the digit is 5 or greater, round up.

24.625

2 is less than 5.
Do not change the digit.

24.6

Round each number to the underlined place-value position.

1. $4\underline{6}.124$	**2.** $29.\underline{9}15$	**3.** $15.1\underline{7}33$	**4.** $0.1\underline{5}9$
5. $308.\underline{8}62$	**6.** $0.05\underline{6}1$	**7.** $0.\underline{5}77$	**8.** $0.\underline{0}089$
9. $\underline{2}.62$	**10.** $76.05\underline{5}2$	**11.** $12.1\underline{9}03$	**12.** $0.\underline{8}55$
13. $3\underline{3}1.98$	**14.** $0.05\underline{4}9$	**15.** $6.\underline{0}3$	**16.** $\underline{1}73.99$
17. $8\underline{4}.012$	**18.** $0.8\underline{4}6$	**19.** $12.76\underline{4}2$	**20.** $0.\underline{0}62$

Name ______________________________ Date ______________

Study Guide Worksheet 2-5

Mental Math Strategy: Powers of Ten

The exponent in a power of ten is the same as the number of zeros in the number.

Examples

	Powers of Ten
10^0	1
10^1	10
10^2	100
10^3	1,000
10^4	10,000
10^5	100,000

To multiply by a power of ten, move the decimal point to the right the number of places shown by the exponent or the number of zeros. Annex zeros if necessary.

Examples $0.08 \times 10^4 = 800$ Move the decimal point 4 places to the right.

$6.25 \times 1{,}000 = 6{,}250$ Move the decimal point 3 places to the right.

Multiply mentally.

1. 0.8×10

2. 6.12×10^2

3. $8.4 \times 1{,}000$

4. 9.3×10^0

5. 0.06×10^4

6. 4.006×100

7. 0.005×10^3

8. 67.8×10^1

Solve each equation.

9. $x = 89 \times 10{,}000$

10. $2.9 \times 10^3 = n$

11. $y = 24.78 \times 10^2$

12. $0.0004 \times 10^4 = p$

13. $c = 8.5 \times 100$

14. $r = 0.01 \times 10^0$

15. $d = 10{,}000 \times 7.07$

16. $0.014 \times 10^2 = k$

17. $v = 589 \times 10^1$

Glencoe Division, Macmillan/McGraw-Hill

Study Guide Worksheet 2-6

Scientific Notation

A number in scientific notation is written as the product of a number equal to or greater than 1 and less than 10 and a power of ten.

Example Write 254,000,000 in scientific notation.

2.54000000 Move the decimal point to get a number between 1 and 10.

2.54×10^8 The decimal point was moved 8 places. The exponent is 8.

A number in scientific notation can be written in standard form.

Example Write 5.6×10^5 in standard form.

560000. Move the decimal point 5 places to the right.

$5.6 \times 10^5 = 560{,}000$

Write each number in scientific notation.

1. 760 **2.** 8,400 **3.** 17,400 **4.** 900,000

5. 12,000,000 **6.** 64 **7.** 5,130,000 **8.** 189,000,000,000

Write each number in standard form.

9. 9.1×10^4 **10.** 8×10^2 **11.** 1.145×10^5 **12.** 3.06×10^3

13. 2.66×10^7 **14.** 7.5×10^6 **15.** 3.03×10^2 **16.** 8.1×10^8

Glencoe Division, Macmillan/McGraw-Hill

Name ______________________ Date ____________

Study Guide Worksheet 2-9

The Metric System

The metric system is a base-10 system. The meter is the basic unit of length. The liter is the basic unit of capacity. The gram is the basic unit of mass.

Prefix	Meaning	Length	Capacity	Mass
kilo-	1,000	kilometer (km)	kiloliter (kL)	kilogram (kg)
	1	meter (m)	liter (L)	gram (g)
centi-	0.01	centimeter (cm)	centiliter (cL)	centigram (cg)
milli-	0.001	millimeter (mm)	milliliter (mL)	milligram (mg)

Units may be changed by multiplying or dividing by multiples of 10.

Examples

1.543 L = ____ mL
There are 1,000 mL in 1 L.
Multiply 1.543 by 1,000.
1.543 L = 1,543 mL

6,724 g = ____ kg
There are 1,000 g in 1 kg.
Divide 6,724 by 1,000.
6,724 g = 6.724 kg

Complete.

1. 0.6 L = __________ mL

2. 89 L = __________ kL

3. 62.4 kg = __________ g

4. 673 mm = __________ cm

5. 9.2 m = __________ cm

6. 55.2 g = __________ kg

7. 20 km = __________ m

8. 0.6 cm = __________ mm

9. 2.2 kL = __________ L

10. 4.5 g = __________ mg

11. 5,900 mL = __________ L

12. 2.5 m = __________ mm

Name ______________________________ Date ______________

Study Guide Worksheet 2-10

Problem-Solving Strategy: Determine Reasonable Answers

Hot dogs cost $3.89 for a package of 8. Hot dog buns cost $1.29 for a package of 6. Kathy thinks the $30 she has is enough to pay for 48 hot dogs and 48 buns. Does this seem reasonable? Explain.

Explore What do you know?
You know that 8 hot dogs cost $3.89 and 6 buns cost $1.29.

You know Kathy wants to buy 48 hot dogs and 48 buns.
What do you want to know?
You want to know if $30 is enough to pay for 48 hot dogs and 48 buns.

Plan Estimate to find the cost of 48 hot dogs and 48 buns.
Then decide if $30 is enough.

Solve Find the number of packages of hot dogs needed. $48 \div 8 = 6$
Round $3.89 to $4. $4 × 6 = $24

Find the number of packages of buns needed. $48 \div 6 = 8$
Round $1.29 to $1. $1 × 8 = $8
$24 + $8 = $32. $30 is not enough.

Examine Since $3.89 is closer to $4 than $1.29 is to $1,
$32 is a low estimate of the amount of money needed.
$30 is not a reasonable amount to expect to pay.

Solve using any strategy.

1. 478 people are taking buses to a rally. Each bus carries 37 people. Thirteen buses have been ordered. Is that a reasonable number?

2. The Ortiz family is planning a 1,880-mile trip. They want to drive between 200 and 250 miles per day. Is 6 days a reasonable time for the trip?

3. How many 60 mg jars can be filled with 1.2 kilograms of spice?

4. Postcards cost $1.89 for 6. If you have $5, can you buy 15 postcards?

5. It is 3:30 P.M. If it takes you 50 minutes to get ready and $1\frac{1}{2}$ hours to get to the airport can you catch a 6 P.M. flight?

6. Amy is 3 years old. Her father is 9 times her age plus 1 year. How old is her father?

Name ______________________ Date ______________

Study Guide Worksheet 4-1

Divisibility Patterns

The following rules will help you determine if a number is divisible by 2, 3, 4, 5, 6, 9, or 10.

A number is divisible by:

- 2 if the digit in the ones place is even.
- 3 if the sum of the digits is divisible by 3.
- 4 if the number formed by the last two digits is divisible by 4.
- 5 if the digit in the ones place is 0 or 5.
- 6 if the number is divisible by both 2 and 3.
- 9 if the sum of the digits is divisible by 9.
- 10 if the digit in the ones place is 0.

Example Determine whether 2,346 is divisible by 2, 3, 4, 5, 6, 9, or 10.

2: The ones digit is 6, an even number.
So 2,346 is divisible by 2.

3: The sum of the digits, $2 + 3 + 4 + 6 = 15$, is divisible by 3.
So 2,346 is divisible by 3.

4: The number formed by the last two digits, 46, is not divisible by 4.
So 2,346 is not divisible by 4.

5: The ones digit is not 0 or 5.
So 2,346 is not divisible by 5.

6: The number is divisible by 2 and by 3.
So 2,346 is divisible by 6.

9: The sum of the digits, 15, is not divisible by 9.
So 2,346 is not divisible by 9.

10: The ones digit is not 0.
So 2,346 is not divisible by 10.

2,346 is divisible by 2, 3, and 6.

Determine whether the first number is divisible by the second number.

1. 65; 5 **2.** 2,641; 3 **3.** 6,780; 10 **4.** 4,185; 9

5. 4,889; 2 **6.** 8,826; 4 **7.** 60,003; 6 **8.** 642; 4

Determine whether each number is divisible by 2, 3, 4, 5, 6, 9, or 10.

9. 660 **10.** 5,025 **11.** 5,091 **12.** 356

Glencoe Division, Macmillan/McGraw-Hill

Name ______________________________ Date ______________

Study Guide Worksheet 4-2

Prime Factorization

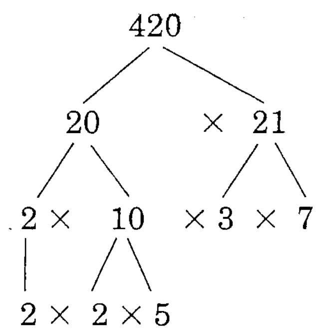
A prime number is a whole number greater than 1 that has **exactly** two factors, 1 and itself.

Examples 7 factors: 1, 7

23 factors: 1, 23

A composite number is a whole number greater than 1 that has more than two factors. Every composite number can be written as the product of prime numbers. This is called the prime factorization of the number.

Example Write the prime factorization of 420.



Write 420 as the product of two factors.

Keep factoring until all of the factors are prime numbers.

The prime factorization of 420 is $2 \times 2 \times 3 \times 5 \times 7$, or $2^2 \times 3 \times 5 \times 7$.

Determine whether each number is composite or prime.

1. 34 **2.** 77 **3.** 37 **4.** 89

5. 69 **6.** 67 **7.** 123 **8.** 71

Write the prime factorization of each number.

9. 490 **10.** 225 **11.** 1,155 **12.** 1,105

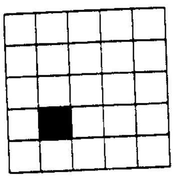
Name ______________________ Date ____________

Practice Worksheet 9-9

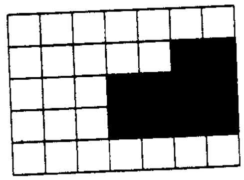
Area Models and Probability

Find the probability that a randomly-dropped counter will fall in the shaded region.

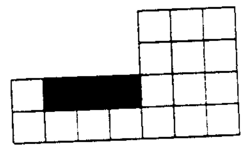
1.



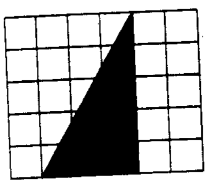
2.



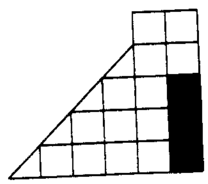
3.



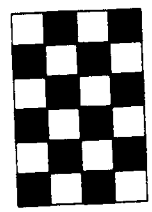
4.



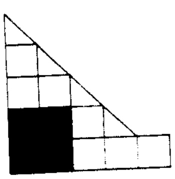
5.



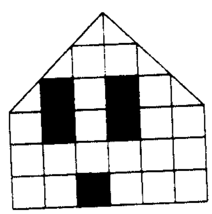
6.



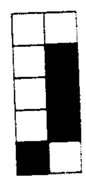
7.



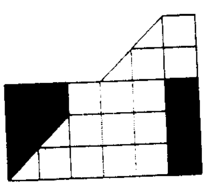
8.



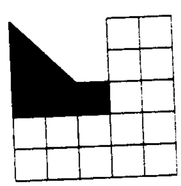
9.



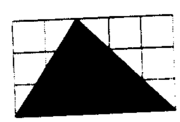
10.



11.



12.



13. Draw a square 4 units on a side on a piece of grid paper. Shade in 12 squares. What is the probability that a randomly-dropped counter will fall in the shaded area?

14. Draw a square 6 units on a side on a piece of grid paper. Shade in 14 squares. What is the probability that a randomly-dropped counter will fall in the shaded area?

Name ____________________ Date ____________

Study Guide Worksheet 4-5

Greatest Common Factor

The greatest common factor (GCF) of two or more numbers is the greatest number that is a factor of each number. One way to find the GCF is to list the factors of each number and then choose the greatest of the common factors.

Example Find the GCF of 72 and 108.

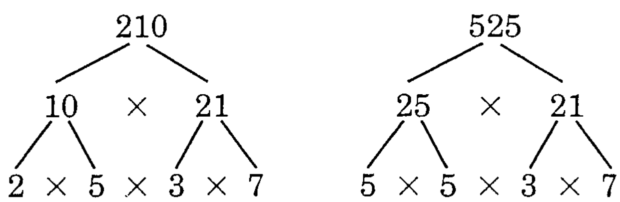
factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
factors of 108: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108

common factors: 1, 2, 3, 4, 6, 9, 12, 18, 36

The GCF of 72 and 108 is 36.

Another way to find the GCF is to write the prime factorization of each number. Then identify all common prime factors and find their product.

Example Find the GCF of 210 and 525.



common prime factors: 3, 5, 7.

The GCF of 210 and 525 is $3 \times 5 \times 7$, or 105.

Find the GCF of each set of numbers.

1. 18, 30

2. 60, 45

3. 24, 72

4. 32, 48

5. 100, 30

6. 54, 36

7. 120, 200

8. 81, 153

9. 77, 121

10. 60, 24, 72

11. 32, 48, 80

12. 90, 120, 180

Name ______________________ Date ______________

Study Guide Worksheet 4-6

Fractions in Simplest Form

A fraction is in simplest form when the greatest common factor (GCF) of the numerator and the denominator is 1.

Example Express $\frac{36}{54}$ in simplest form.

factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

The GCF of 36 and 54 is 18.

$\frac{36}{54} = \frac{36 \div 18}{54 \div 18} = \frac{2}{3}$ Divide the numerator and denominator by the GCF.

The GCF of 2 and 3 is 1.

The simplest form of $\frac{36}{54}$ is $\frac{2}{3}$.

Express each fraction in simplest form.

1. $\frac{30}{72}$ **2.** $\frac{45}{60}$ **3.** $\frac{68}{84}$

4. $\frac{54}{66}$ **5.** $\frac{56}{64}$ **6.** $\frac{17}{119}$

7. $\frac{60}{75}$ **8.** $\frac{75}{375}$ **9.** $\frac{36}{48}$

10. $\frac{33}{132}$ **11.** $\frac{450}{750}$ **12.** $\frac{25}{125}$

13. $\frac{36}{81}$ **14.** $\frac{32}{42}$ **15.** $\frac{77}{88}$

Name ______________________ Date ______________

Practice Worksheet 2-10

Problem-Solving Strategy: Determine Reasonable Answers

Solve. Then determine if the answer is reasonable.

1. The Kowalski family spent $1,500 on a one-week vacation. Their calculator showed an average cost of $150 per day. Is this answer reasonable. Explain.

2. Compact Disc Depot is selling CD's for $9.98. Carrie thinks she can buy 5 CD's for less than $50. Is her answer reasonable?

Solve using any strategy.

3. Martin's mother promised him $1.10 for each mile he rode in a bike-a-thon. Martin rode 22 miles. Did he earn about $24.00 or $240.00?

4. Suppose the price of a $300 video recorder is reduced by $80. During a special promotion, an additional discount of $15 is offered. What is the final price during the promotion?

5. An automobile manufacturer advertises that its highest-mileage car gets 55 miles per gallon of gasoline. If the gasoline tank holds 11 gallons, what is a reasonable estimate of how far the car can travel on one tank of gasoline?

6. Suppose you take $20 to the store to buy school supplies. You choose 4 notebooks at $1.98 each, 2 pens at $0.89 each, 5 packages of notebook paper at $1.50 each, an eraser at $0.39, and 4 pencils at $0.10 each. Did you have enough money?

7. Suppose a long distance telephone call costs $0.20 for the first minute plus $0.10 for each additional minute. How much does a 15-minute call cost?

8. During the 1990–1991 basketball season, Michael Jordan scored an average of 31.5 points for each of 82 games. What is a reasonable estimate of the total number of points he scored during the season?

Glencoe Division, Macmillan/McGraw-Hill

NAME ______________________________ DATE ______________

9-1 Study Guide

Ratios and Rates

Student Edition
Pages 432–436

A **ratio** is a comparison of two numbers by division. If the two terms of a ratio have no common factors, the ratio is in simplest form.

$$\frac{4}{16} \overset{\div 4}{=} \frac{1}{4}$$ The GCF of 4 and 16 is 4.

One type of ratio is a **rate**. A rate compares two measurements with different units. Speeds, such as 50 miles per hour or 32 feet per second, are familiar examples of rates. To change a rate to a unit rate, divide both the numerator and denominator by the denominator.

Rate → Unit Rate

$$\frac{5280 \text{ ft}}{4 \text{ min}} \overset{\div 4}{=} \frac{1320 \text{ ft}}{1 \text{ min}}$$ A unit rate is a rate with a denominator of 1.

Write the ratio that compares each of the following.

1. number of *p*'s to number of *i*'s in *Mississippi*
2. number of *o*'s to total number of letters in *proportion*
3. number of months that have an *r* in their name to the number of months in a year

Express each ratio or rate as a fraction in simplest form.

4. 9 to 12
5. 12 to 9
6. 5:20
7. $2.50 for 5 notepads
8. 60¢ per dozen
9. $3.00 to rent 2 videotapes

Express each ratio as a unit rate.

10. 120 miles in 2 hours
11. 800 pounds for 40 square inches
12. $300 for 5 jackets
13. 45 meters in 3 minutes
14. 10 kilometers in 2 hours
15. 30 yards in 15 seconds

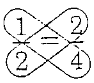
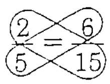
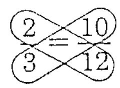
NAME ______________________________ DATE ____________

9-4 Study Guide
Using Proportions

Student Edition
Pages 444–447

A proportion is a statement of equality of two or more ratios. To determine if two ratios form a proportion, check their cross products. If the cross products are equal, the rations form a proportion.

$\frac{1}{2} \stackrel{?}{=} \frac{2}{4}$ $\qquad$ $\frac{2}{5} \stackrel{?}{=} \frac{6}{15}$ $\qquad$ $\frac{2}{3} \stackrel{?}{=} \frac{10}{12}$

$1 \times 4 \stackrel{?}{=} 2 \times 2$
$4 = 4$ ✓ It is a proportion.

$2 \times 15 \stackrel{?}{=} 5 \times 6$
$30 = 30$ ✓ It is a proportion.

$2 \times 12 \stackrel{?}{=} 3 \times 10$
$24 \neq 30$ It is not a proportion.

Cross products can be used to solve proportions.

Example: Solve the proportion $\frac{3}{4} = \frac{x}{20}$.

$3 \cdot 20 = 4 \cdot x$	Write the cross products.
$60 = 4x$	Multiply.
$15 = x$	Divide each side by 4.

Write = or ≠ in each blank to make a true statement.

1. $\frac{3}{8}$ ____ $\frac{12}{32}$
2. $\frac{15}{20}$ ____ $\frac{3}{4}$
3. $\frac{4}{7}$ ____ $\frac{16}{49}$
4. $\frac{1}{2}$ ____ $\frac{1}{4}$
5. $\frac{35}{50}$ ____ $\frac{7}{10}$
6. $\frac{40}{48}$ ____ $\frac{5}{6}$

Solve each proportion.

7. $\frac{5}{8} = \frac{x}{40}$
8. $\frac{6}{3} = \frac{10}{t}$
9. $\frac{n}{5} = \frac{42}{7}$
10. $\frac{4}{11} = \frac{12}{x}$
11. $\frac{2}{3} = \frac{0.8}{n}$
12. $\frac{7}{12} = \frac{1.68}{b}$

Write a proportion that could be used to solve each problem. Then solve the proportion.

13. Cole can pick 2 rows of beans in 30 minutes. How long will it take him to pick 5 rows if he works at the same rate?

14. A tree casts a shadow 30 meters long. A 2.8-meter pole casts a shadow 2 meters long. How tall is the tree?

Study Guide

Variables and Expressions

Any letter used to represent an unspecified number is called a variable. You can use variables to translate verbal expressions into algebraic expressions.

Words	Symbols
4 more than a number	$x + 4$
a number decreased by 8	$b - 8$
the product of 5 and a number	$5c$
a number divided by 8	$h \div 8$ or $\frac{h}{8}$
a number squared	y^2

The algebraic expression x^n represents a product in which each factor is the same. The small raised n is the exponent and it tells how many times the base, x, is used as a factor.

Example: Evaluate 3^4.

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$$
$$= 81$$

Write a verbal expression for each algebraic expression.

1. $w - 1$ **2.** $\frac{1}{3}a^3$ **3.** $81 + 2x$

Write an algebraic expression for each verbal expression.

4. a number decreased by 5

5. four times a number

6. 8 less than a number

7. a number divided by 6

8. a number multiplied by 37

9. the sum of a number and 9

10. 3 less than 5 times a number

11. twice the sum of 15 and a number

12. 7 more than the product of 6 and a number

13. 30 increased by 3 times the square of a number

Write each expression as an expression with exponents.

14. $7 \cdot 7 \cdot 7$ **15.** $3 \cdot p \cdot p$ **16.** $9(b)(b)(b)(b)(b)$

Evaluate each expression.

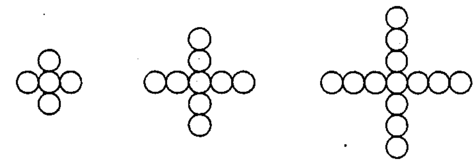
17. 2^3 **18.** 10^5 **19.** 4^4

Study Guide

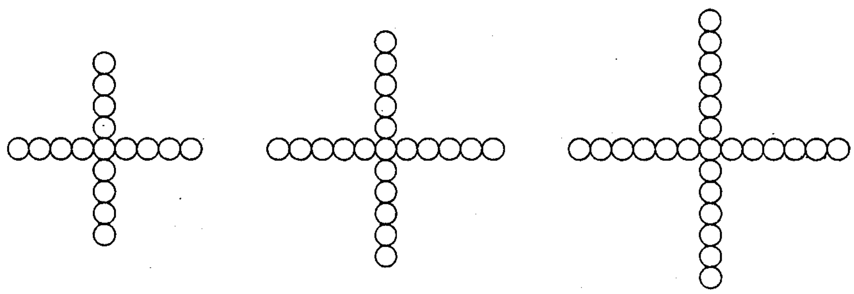
Patterns and Sequences

When solving certain problems, you must often look for a pattern. A **pattern** is a repeated design or arrangement.

Example 1: Study the pattern at the right.



Draw the next three figures in the pattern. The pattern begins by adding one circle to all four sides of the original, then continues by adding an extra circle to all four sides. The next three figures are drawn below.



The numbers 1, 3, 5, 7, 9 and 11 form a **sequence.** A sequence is a set of numbers in a specific order. The numbers in a sequence are called **terms.**

Example 2: Find the next three numbers in each sequence.

a. 2, 6, 18, 54, ⋯ Study the pattern in the sequence. Each term is 3 times more than the term before it. The next three terms are 162, 486, and 1,458.

b. 128, 64, 32, ⋯ Study the pattern in the sequence. Each term is $\frac{1}{2}$ of the term before it. The next three terms are 16, 8, and 4.

Give the next two items for each pattern.

1. 

2. 

3. What color is the 50th figure in Exercise 2? Explain your reasoning.

4. 2, 12, 72, 432, ⋯

5. $b + 1, b + 4, b + 9, b + 16, \cdots$

6. 10, 7, 11, 8, 12, 9, 13, ⋯

NAME ______________________ DATE ________

Study Guide

Order of Operations

Ian said Shelley won first prize and I won second prize. Without punctuation, this sentence has three possible meanings.

Ian said, "Shelley won first prize and I won second prize."

or

"Ian," said Shelley, "won first prize and I won second prize."

or

Ian said Shelley won first prize and I [that is, the speaker] won second prize.

In mathematics, in order to avoid confusion about meaning, an agreed-upon order of operations tells us whether a mathematical expression such as $15 - 12 \div 4$ means $(15 - 12) \div 4$ or $15 - (12 \div 4)$. That order is shown at the right.

Order of Operations
1. Simplify expressions inside grouping symbols. 2. Evaluate all powers. 3. Do all multiplications and divisions from left to right. 4. Do all additions and subtractions from left to right.

You can evaluate an algebraic expression when the value of each variable is known. Replace each variable with its value and then use the order of operations to perform the indicated operations. Remember to do all operations within grouping symbols first.

Example 1: Evaluate $15 - 12 \div 4$.

$$\begin{aligned} 15 - 12 \div 4 &= 15 - 3 \\ &= 12 \end{aligned}$$

Example 2: Evaluate $x^3 + 5(y - 3)$ if $x = 2$ and $y = 12$.

$$\begin{aligned} x^3 + 5(y - 3) &= 2^3 + 5(12 - 3) \\ &= 2^3 + 5(9) \\ &= 8 + 5(9) \\ &= 8 + 45 \\ &= 53 \end{aligned}$$

Evaluate each expression.

1. $10 + 8 \cdot 1$

2. $3^2 \div 3 + 2^2 \cdot 7 - 20 \div 5$

3. $12(20 - 17) - 3 \cdot 6$

4. $\dfrac{15 + 60}{30 - 5}$

Evaluate each expression when $x = 2$, $y = 3$, $a = \frac{4}{5}$, and $b = \frac{3}{5}$.

5. $x + 7$

6. $3x - 5$

7. $6a + 8b$

8. $a^2 + 2b$

9. $\dfrac{5\,a^2b}{y}$

10. $(10x)^2 + 100a$

11. $23 - (a + b)$

12. $\dfrac{x^4 - y^2}{3ay}$

NAME ______________________ DATE __________

Study Guide

Student Edition
Pages 45–50

The Distributive Property

When you find the product of two integers, you find the sum of two partial products. For example, you can write

$$\begin{array}{r} 58 \\ \times\ 5 \\ \hline 290 \end{array} \quad \text{as} \quad \begin{array}{r} 50 + 8 \\ \times \quad\ 5 \\ \hline 250 + 40 \end{array} \leftarrow (50 \times 5) + (8 \times 5)$$

The statement $(50 + 8) \times 5 = (50 \times 5) + (8 \times 5)$ illustrates the distributive property. The multiplier 5 is distributed over the 50 and the 8.

Distributive Property
For any numbers a, b, and c, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$; $a(b - c) = ab - ac$ and $(b - c)a = ba - bc$.

You can use the distributive property to simplify algebraic expressions.

Example: Simplify $4(a^2 + 3ab) - ab$.

$4(a^2 + 3ab) - ab$	$= 4(a^2 + 3ab) - 1ab$	**Multiplicative identity**
	$= 4a^2 + 12ab - 1ab$	**Distributive property**
	$= 4a^2 + (12 - 1)ab$	**Distributive property**
	$= 4a^2 + 11ab$	**Substitution**

Name the coefficient of each term. Then name the like terms in each list of terms.

1. $3x, 3x^2, 5x$

2. $2mn, 10mn^2, 12mn^2, mn^2$

Use the distributive property to find each product.

3. $4 \cdot 315$

4. $3 \cdot 24$

Use the distributive property to rewrite each expression.

5. $5(4x - 9)$

6. $9r^2 + 9s^2$

Simplify each expression, if possible. If not possible, write in simplest form.

7. $7b + 3b$

8. $4(5ac - 7)$

9. $21c + 18c + 31b - 3b$

10. $10x^2 - 6x^3$

11. $10xy - 4(xy + xy)$

12. $0.2(0.8 + 7y) + 0.36y$

NAME ______________________ DATE ____________

Study Guide

Student Edition
Pages 51–55

Commutative and Associative Properties

The commutative and associative properties can be used to simplify expressions.

Commutative Properties
For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.

Associative Properties
For any numbers a, b, and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

Example: Simplify $8(y + 2x) + 7y$.

$8(y + 2x) + 7y = (8y + 16x) + 7y$	**Distributive property**
$= (16x + 8y) + 7y$	**Commutative property of addition**
$= 16x + (8y + 7y)$	**Associative property of addition**
$= 16x + (8 + 7)y$	**Distributive property**
$= 16x + 15y$	**Substitution property of equality**

Simplify.

1. $4x + 3y + x$

2. $8r^2s + 2rs^2 + 7r^2s$

3. $6(2x + 4y) + 2(x + 9)$

4. $3a^2 + 4b + 10a^2$

5. $4xy + 7x^2y + xy$

6. $3ab + 4a^2b + 5(2a^2b)$

7. $6(a + b) - a + 3b$

8. $0.5(18x + 16y) + 13x$

9. $\frac{2}{3} + \frac{1}{2}(x + 10) + \frac{4}{3}$

10. $5(0.3x + 0.1y) + 0.2x$

11. $5(2y + 3x) + 6(y + x)$

12. $z^2 + 9x^2 + \frac{4}{3}z^2 + \frac{1}{3}x^2$

Name the property illustrated by each statement.

13. $6x + 2y = 2y + 6x$

14. $15(a + 4) = 15a + 15(4)$

15. $1 \cdot b^3 = b^3$

16. $(2c + 6) + 10 = 2c + (6 + 10)$

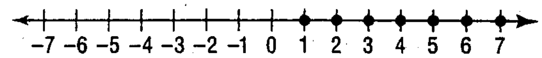
NAME ______________________ DATE ________

Study Guide

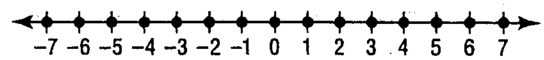
Student Edition
Pages 72–77

Integers and the Number Line

The figure at the right is part of a number line. On a number line, the distances marked to the right of 0 are named by members of the set of **whole numbers.**

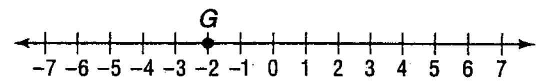


The set of numbers used to name the points marked on the number line at the right is called the set of **integers.**



To graph a set of numbers means to locate the points named by those numbers on the number line. The number that corresponds to a point on the number line is called the **coordinate** of the point.

Name the coordinate of point G.

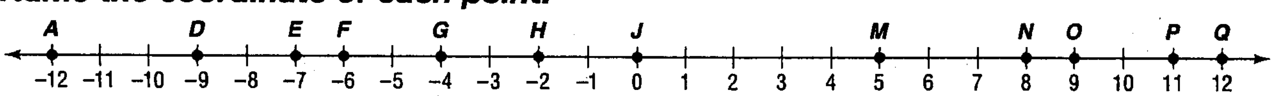


The coordinate of G is -2.

A number line is often used to show addition of integers. For example, to find the sum of 3 and -5, follow the steps at the right.

Step 1 Draw an arrow, starting at 0 and going to 3.
Step 2 Start at 3. Draw an arrow 5 units to the left.
Step 3 The second arrow points to the sum, −2.

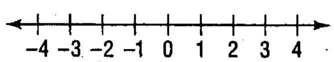
Name the coordinate of each point.



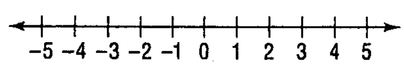
1. M	**2.** Q	**3.** H	**4.** E	**5.** J	**6.** A
7. G	**8.** P	**9.** F	**10.** N	**11.** O	**12.** D

Graph each set of numbers on a number line.

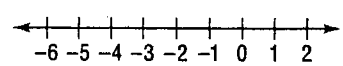
13. {−3, −1, 1, 3}



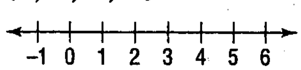
14. {−5, −2, 1, 4}



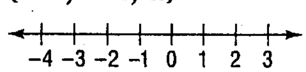
15. {integers less than 0}



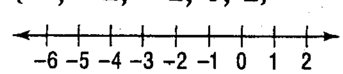
16. {0, 1, 3, 5}



17. {−3, −2, 2}



18. {⋯, −2, −1, 0, 1}



Find each sum. If necessary, use a number line.

19. $2 + 3$

20. $9 + 1$

21. $-5 + (-1)$

22. $-10 + 6$

23. $9 + (-9)$

24. $0 + (-4)$

25. $-8 + (-3)$

26. $6 + (-10)$

27. $-6 + 6$

NAME ______________________________ DATE ______________

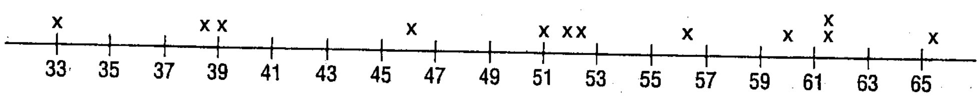
Study Guide

Integration: Statistics
Line Plots

The table to the right shows the final results of the Great Frog Competition in Dickerson County. The frog jumps were measured in inches.

	Jumps		
Frogs	1	2	3
Slippery	61.5	51.9	60.0
Spots	46.0	38.5	39.1
Inky	56.2	33.0	61.5
Popper	65.4	51.0	52.3

Numerical information displayed on a number line is called a **line plot.** The line plot below is another way to show the data for the frog jumping contest.



1. Make a table comparing the highest mountains in the United States: Mt. McKinley, 20,633 ft; Mt. Elbert, 14,433 ft; Mt. Rainier, 14,410 ft; Humphrey Peak, 12,633 ft; Kings Peak, 13,528 ft.

Mountain	Height (feet)

Use the table at the right for Exercises 2–4.

2. Make a line plot representing the weights of wrestlers. Insert labels and "x"s on the number line below.

Weights of Junior Varsity Wrestlers (pounds)				
170	160	135	135	160
122	188	154	108	135
140	122	103	190	154

3. How many wrestlers are in weight classes over 140 lb?

4. What is the greatest weight class?

Adding and Subtracting Integers

Use the following definitions, rules, and properties when adding or subtracting integers.

Definition, Rule, or Property		Example
Definition of Absolute Value	For any real number a: if $a > 0$, then $\|a\| = a$, and if $a < 0$, then $\|a\| = -a$. $\|0\| = 0$	$\|2\| = 2$ $\|-2\| = 2$
Adding Integers with the Same Sign	To add integers with the same sign, add their absolute values. Give the sum the same sign as the addends.	$3 + 2 = 5$ $-3 + (-2) = -5$
Adding Integers with Different Signs	To add integers with different signs, subtract the lesser absolute value from the greater absolute value. Give the result the same sign as the addend with the greater absolute value.	$-7 + 6 = -1$ $8 + (-4) = 4$
Additive Inverse Property	For every number a, $a + (-a) = 0$.	$-9 + 9 = 0$
Subtraction Rule	To subtract a number, add its additive inverse. For any numbers a and b, $a - b = a + (-b)$.	$8 - (-2) = 8 + 2$ $= 10$

You can use the distributive property and the addition and subtraction rules for integers to simplify expressions with like terms.

Example: Simplify $-6x - x + 9x$.

$$\begin{aligned} -6x - x + 9x &= -6x + (-1x) + 9x \\ &= [-6 + (-1) + 9]x \\ &= (-7 + 9)x \\ &= 2x \end{aligned}$$

Find each sum or difference.

1. $-17 + (-16)$

2. $107 + (-40)$

3. $75 + 86$

4. $11 - 41$

5. $15 - (-21)$

6. $-33 - (-17)$

7. $3m + (-15m) - 11m$

8. $-6a + 15a + (-11a)$

9. $-9y + 20y - (-6y)$

Evaluate each expression if $x = -4$, $y = 3$, and $z = -7$.

10. $456 + |z|$

11. $z + (-71) + |y|$

12. $-11 - |x|$

13. $31 - y - |x|$

NAME ________________________ DATE ________

Study Guide

Student Edition
Pages 93–99

Rational Numbers

Definition of a Rational Number	A rational number is a number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

You can compare rational numbers by graphing them on a number line.

Comparing Numbers on the Number Line	If a and b represent any numbers and the graph of a is to the left of the graph of b, then $a < b$. If the graph of a is to the right of the graph of b, then $a > b$.
Comparison Property	For any two numbers a and b, exactly one of the following sentences is true. $a < b$ $a = b$ $a > b$

Example 1: $-3\frac{1}{2} < -\frac{1}{2}$ The graph of $-3\frac{1}{2}$ is to the left of the graph of $-\frac{1}{2}$.

Example 2: $-2\frac{1}{4} > -3\frac{1}{4}$ The graph of $-2\frac{1}{4}$ is to the right of the graph of $-3\frac{1}{4}$.

Example 3: Replace __?__ with <, >, or = to make the sentence true.

-15 __?__ -3

$-15 < -3$ Since -15 is to the left of -3 on a number line, -15 is less than -3.

The symbols $\neq$, $\leq$ and $\geq$ can also be used to compare numbers and are called **inequality symbols.**

You can use **cross products** to compare two fractions with different denominators.

Comparison Property for Rational Numbers	For any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, with $b > 0$ and $d > 0$: 1. if $\frac{a}{b} < \frac{c}{d}$ then $ad < bc$, and 2. if $ad < bc$, then $\frac{a}{b} < \frac{c}{d}$.

This property also holds if < is replaced by >, $\leq$, $\geq$, or =.

A property that is true for rational numbers but is not true for integers is the **density property.**

Density Property for Rational Numbers	Between every pair of distinct rational numbers, there is another rational number.

Replace each __?__ with <, >, or = to make each sentence true.

1. -4 __?__ 10

2. $\frac{-29}{2}$ __?__ $-28.5 + 14$

3. $-5 - 6$ __?__ $-12 - 1$

Write the numbers in each set in order from least to greatest.

4. $3\frac{1}{3}, \frac{5}{8}, 0.4$

5. $-\frac{3}{2}, \frac{1}{4}, 0.2$

Find a number between the given numbers.

6. $\frac{1}{2}$ and $\frac{7}{9}$

7. $\frac{7}{6}$ and $\frac{9}{8}$

8. $\frac{9}{17}$ and $\frac{2}{5}$

NAME ____________________ DATE ________

Study Guide

Student Edition
Pages 100–104

Adding and Subtracting Rational Numbers

The rules for adding and subtracting integers also apply to adding and subtracting rational numbers.

Rational Number	Form $\frac{a}{b}$
3	$\frac{3}{1}$
$-2\frac{3}{4}$	$-\frac{11}{4}$
0.125	$\frac{1}{8}$

Example 1: Add $\left(-2\frac{1}{3}\right) + 5\frac{2}{3}$.

$$\left(-2\frac{1}{3}\right) + 5\frac{2}{3} = +\left(\left|5\frac{2}{3}\right| - \left|-2\frac{1}{3}\right|\right)$$
$$= +\left(5\frac{2}{3} - 2\frac{1}{3}\right)$$
$$= 3\frac{1}{3}$$

Example 2: Subtract $-3.42 - 5.82$.

$$-3.42 - 5.82 = -3.42 + (-5.82)$$
$$= -9.24$$

Previously you have added pairs of numbers. To add three or more numbers, first group the numbers in pairs. Use the commutative and associative properties to rearrange the addends if necessary. Study the example at the right.

Example 3: Add $-\frac{2}{3} + \frac{4}{5} + \left(-\frac{5}{3}\right)$.

$$-\frac{2}{3} + \frac{4}{5} + \left(-\frac{5}{3}\right) = \left[-\frac{2}{3} + \left(-\frac{5}{3}\right)\right] + \frac{4}{5}$$
$$= -\frac{7}{3} + \frac{4}{5}$$
$$= -\frac{35}{15} + \frac{12}{15}$$
$$= -\frac{23}{15} \text{ or } -1\frac{8}{15}$$

Find each sum or difference

1. $-\frac{9}{11} + \left(-\frac{13}{11}\right)$ **2.** $\frac{5}{8} + \left(-\frac{1}{12}\right)$ **3.** $-0.005 + 0.0043$

4. $\frac{3}{8} - \left(-\frac{1}{8}\right)$ **5.** $4.59 - 2.31$ **6.** $-\frac{7}{5} - \frac{2}{7}$

Evaluate each expression if x = −4, y = 3, z = −7.

7. $x + 16$ **8.** $0 + y$ **9.** $27 - (x - z)$ **10.** $100 + (x + y)$

Find each sum.

11. $-36.4 + 29.15 + (-14.2)$ **12.** $6.5x + 12.3x + (-14.9x)$ **13.** $0.85 + 13.6 + (-3.01)$

14. $-9y + (-20y) + 6y$ **15.** $\frac{3}{5} + \left(-\frac{5}{8}\right) + \frac{1}{4}$ **16.** $12p + 11p + (-23p)$

NAME ________________________ DATE ____________

Study Guide

Student Edition
Pages 106–111

Multiplying Rational Numbers

You can use the rules below when multiplying rational numbers.

Rule or Property		Example
Multiplying Two Numbers with Different Signs	The product of two numbers that have different signs is negative.	$(-2x)5y = (-2)(5)xy$ $= -10xy$
Multiplying Two Numbers with the Same Sign	The product of two numbers that have the same sign is positive.	$(-4)(-7) = 28$
Multiplicative Property of −1	The product of any number and −1 is its additive inverse. $-1(a) = -a$ and $a(-1) = -a$	$(-3)(-4)(-1)(2) = 12(-1)(2)$ $= -12(2)$ $= -24$

To find the product of two or more numbers, first group the numbers in pairs.

Example: Multiply $(-2.3)(5.6)(-0.7)(-0.2)$

$$\begin{aligned}(-2.3)(5.6)(-0.7)(-0.2) &= [(-2.3)(5.6)][(-0.7)(-0.2)] && \text{Associative property } (\times)\\ &= (-12.88)(0.14) && \text{Substitution property } (=)\\ &= -1.8032\end{aligned}$$

Find each product.

1. $(-24)(-2)$

2. $(6.0)(-0.3)$

3. $(-2)(-3)(-4)$

4. $\left(\frac{1}{2}\right)(-10)(5)$

5. $(-22)(-3)\left(-\frac{2}{3}\right)$

6. $\left(\frac{4}{5}\right)(-5)(0)(4)$

Simplify.

7. $(-6)(5) + (2)(4)$

8. $\left(-\frac{3}{4}\right)\left(\frac{1}{8}\right) - \left(\frac{1}{4}\right)\left(\frac{1}{8}\right)$

9. $-5(2x + x) - 3(-xy)$

10. $(-5ab)6 - ab(4 + 1)$

11. $2.3(4c - d) - 0.1(-0.5c + 8d)$

12. $(-0.9)(1.1) - (-5.1)(0.6)$

NAME ______________________ DATE ________

Study Guide

Dividing Rational Numbers

Use the following rules to divide rational numbers.

Rule or Property		Example
Dividing Rational Numbers	The quotient of two numbers is positive if the numbers have the same sign. The quotient of two numbers is negative if the numbers have different signs.	$-60 \div (-10) = 6$ $-48 \div 4 = -12$
Multiplicative Inverse Property	For every nonzero number a, there is exactly one number $\frac{1}{a}$, such that $(a)\frac{1}{a} = \frac{1}{a}(a) = 1$.	$\frac{1}{3} \div \frac{3}{4} = \frac{1}{3} \cdot \frac{4}{3}$ $= \frac{4}{9}$
Division Rule	For all numbers a and b, with $b \neq 0$, $a \div b = \frac{a}{b} = a\left(\frac{1}{b}\right) = \frac{1}{b}(a)$.	$6 \div 2 = \frac{6}{2}$ $= (6)\frac{1}{2}$ $= \frac{1}{2}(6) = 3$

Since the fraction bar indicates division, you can use the division rules and the distributive property to simplify rational expressions.

Example: Simplify $\frac{-20a + 15}{5}$.

$$\frac{-20a + 15}{5} = (-20a + 15)\left(\frac{1}{5}\right)$$
$$= (-20a)\left(\frac{1}{5}\right) + (15)\left(\frac{1}{5}\right)$$
$$= -4a + 3$$

Simplify.

1. $\frac{-44a}{4}$

2. $\frac{16x}{2}$

3. $\frac{80}{5}$

4. $\frac{81}{-27}$

5. $\frac{-144a}{6}$

6. $\frac{-30}{-10} \div \frac{30}{10}$

7. $\frac{57y}{3}$

8. $-\frac{1}{2} \div 8$

9. $\frac{18a - 6b}{-3}$

10. $\frac{12x}{3} \div \frac{1}{12} + xyz$

11. $\frac{36a - 12}{12}$

12. $\frac{-\frac{5}{8}}{5}$

NAME________________________________ DATE ____________

Study Guide

Student Edition
Pages 144–149

Solving Equations with Addition and Subtraction

You can use the addition and subtraction properties of equality to solve equations. To check, substitute the solution for the variable in the original equation. If the resulting sentence is true, your solution is correct.

Addition Property of Equality	For any numbers a, b, and c, if $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	For any numbers a, b, and c, if $a = b$, then $a - c = b - c$.

Example 1: Solve $r - 6 = -11$.

$$r - 6 = -11$$
$$r - 6 + 6 = -11 + 6$$
$$r = -5$$

Check:

$$r - 6 = -11$$
$$-5 - 6 = -11$$
$$-11 = -11 \checkmark$$

Example 2: Solve $k + 18 = -9$.

$$k + 18 = -9$$
$$k + 18 - 18 = -9 - 18$$
$$k = -27$$

Check:

$$k + 18 = -9$$
$$-27 + 18 = -9$$
$$-9 = -9 \checkmark$$

Sometimes an equation can be solved more easily if it is rewritten first. Recall that subtracting a number is the same as adding its inverse. For example, the equation $g - (-5) = 18$ may be rewritten as $g + 5 = 18$.

Solve each equation. Then check your solution.

1. $b - 17 = -40$

2. $x + 12 = 6$

3. $z + 2 = -13$

4. $-17 = b + 4$

5. $s + (-9) = 7$

6. $v - (-12) = 10$

7. $19 + h = -4$

8. $73 = 29 - q$

9. $-3.2 = l + (-0.2)$

10. $-25 - r = \frac{4}{36}$

11. $-\frac{3}{8} + x = \frac{5}{8}$

12. $\frac{5}{9} = -y + \frac{2}{15}$

NAME______________________ DATE __________

Study Guide

Solving Equations with Multiplication and Division

You can solve equations in which a variable has a coefficient by using the multiplication and division properties of equality.

Multiplicative Property of Equality	For any numbers a, b, and c, with $c \neq 0$, if $a = b$, then $ac = bc$.
Division Property of Equality	For any numbers a, b, and c, with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Example 1: Solve $\frac{1}{4}n = 16$.

$$\frac{1}{4}n = 16$$
$$4\left(\frac{1}{4}n\right) = 4(16)$$
$$n = 64$$

Check:
$$\frac{1}{4}n = 16$$
$$\frac{1}{4}(64) \stackrel{?}{=} 16$$
$$16 = 16 \checkmark$$

Example 2: Solve $8n = 64$.

$$8n = 64$$
$$\frac{8n}{8} = \frac{64}{8}$$
$$n = 8$$

Check:
$$8n = 64$$
$$8(8) \stackrel{?}{=} 64$$
$$64 = 64 \checkmark$$

Solve each equation. Then check your solution.

1. $-3r = -24$

2. $8s = -64$

3. $-3t = 51$

4. $\frac{1}{4}w = -16$

5. $6x = \frac{3}{4}$

6. $1\frac{1}{4}y = -3\frac{3}{4}$

Define a variable, write an equation, and solve each problem. Then check your solution.

7. Twelve times a number is 96. What is the number?

8. One half of a number is fifteen. What is the number?

9. Negative four times a number is −112. What is the number?

10. Regina paid $53.50 for 5 basketball tickets. What is the cost of each ticket?

Complete.

11. If $4x = 100$, then $8x =$ ______.

12. If $6y = 36$, then $3y =$ ______.

13. If $-10a = 53$, then $-5a =$ ______.

14. If $2g + h = 12$, then $4g + 2h =$ ______.

NAME ______________________ DATE ________

Study Guide

Solving Multi-Step Equations

When solving some equations you must perform more than one operation on both sides. First, determine what operations have been done to the variable. Then undo these operations in the reverse order.

Example 1: How would you solve $\frac{n}{3} - 7 = 28$?

$$\frac{n}{3} - 7 = 28$$

First, n was divided by 3. Then 7 was subtracted. } To solve, first add 7 to each side. Then multiply each side by 3.

Procedure for Solving a Two-Step Equation	1. Undo any indicated additions or subtractions. 2. Undo any indicated multiplications or divisions involving the variable.

Example 2:

$5x + 3 = 23$	**Addition of 3 is indicated.**	***Check:***
$5x + 3 - 3 = 23 - 3$	**Therefore, subtract 3 from each side.**	$5x + 3 = 23$
$5x = 20$	**Multiplication by 5 is also indicated.**	$5(4) + 3 \stackrel{?}{=} 23$
$\frac{5x}{5} = \frac{20}{5}$	**Therefore, divide each side by 5.**	$20 + 3 \stackrel{?}{=} 23$
$x = 4$		$23 = 23$ ✔

Solve each equation. Then check your solution.

1. $5z + 16 = 51$

2. $14n - 8 = 34$

3. $0.6x - 1.5 = 1.8$

4. $\frac{4b + 8}{-2} = 10$

5. $16 = \frac{d - 12}{14}$

6. $8 + \frac{3n}{12} = 13$

7. $\frac{7}{8}p - 4 = 10$

8. $\frac{g}{-5} + 3 = -13$

9. $-4 = \frac{7x - (-1)}{-8}$

Define a variable, write an equation, and solve each problem. Then check your solution.

10. Find three consecutive integers whose sum is 96.

11. Find two consecutive odd integers whose sum is 176.

NAME ______________________ DATE ________

Study Guide

Student Edition
Pages 168–172

Solving Equations with the Variable on Both Sides

When an equation contains parentheses or other grouping symbols, first use the distributive property to remove the grouping symbols. If the equation has variables on each side, use addition and subtraction property of equality to write an equivalent equation that has all the variables on one side. Then solve the equation.

Example: Solve $4(2a - 1) = -10(a - 5)$.

$$4(2a - 1) = -10(a - 5)$$

$$8a - 4 = -10a + 50$$ **Use the distributive property.**

$$8a + 10a - 4 = -10a + 10a + 50$$ **Add 10*a* to each side.**

$$18a - 4 = 50$$

$$18a - 4 + 4 = 50 + 4$$ **Add 4 to each side.**

$$18a = 54$$

$$\frac{18a}{18} = \frac{54}{18}$$ **Divide each side by 18.**

$$a = 3$$

Check:

$$4(2a - 1) = -10(a - 5)$$

$$4(2 \cdot 3 - 1) = -10(3 - 5)$$

$$4(6 - 1) = -10(-2)$$

$$4(5) = -10(-2)$$

$$20 = 20 \checkmark$$

Some equations may have *no solution,* and some equations may have *every number* in their solution set. An equation that is true for every value of the variable is called an **identity.**

Solve each equation. Then check your solution.

1. $-3(x + 5) = 3(x - 1)$

2. $6 - b = 5b + 30$

3. $5y - 2y = 3y + 2$

4. $2(7 + 3t) = -t$

5. $3(a + 1) - 5 = 3a - 2$

6. $75 - 9g = 5(-4 + 2g)$

7. $1.2x + 4.3 = 2.1 - x$

8. $4.4s + 6.2 = 8.8s - 1.8$

9. $5(f + 2) = 2(3 - f)$

10. $\frac{1}{2}b + 4 = \frac{1}{8}b + 88$

11. $\frac{2}{5}w - w = -\frac{1}{5}(3w + 2)$

12. $5(p + 3) + 9 = 3(p - 2) + 6$

13. $\frac{3}{4}k - 5 = \frac{1}{4}k - 1$

14. $0.03g - (2g + 3) = 1.8$

15. $-5(2r + 3) = 3(11 - 4r) - 58$

NAME ______________________ DATE ________

Study Guide

Ratios and Proportions

In mathematics, a **ratio** compares two numbers by division. A ratio that compares a number a to a number b can be written in the following ways.

a to b $\qquad$ a:b $\qquad$ $\frac{a}{b}$

When a ratio compares two quantities with different units of measure, that ratio is called a **rate.** For example, a 5°C rise in temperature per hour is a rate and can be expressed as $\frac{5 \text{ degrees}}{1 \text{ hour}}$, or 5 degrees per hour.

Proportions are often used to solve problems involving ratios. You can use the means-extremes property of proportions to solve equations that have the form of a proportion.

Definition of Proportion	An equation of the form $\frac{a}{b} = \frac{c}{d}$ stating that two ratios are equal is called a proportion.
Means-Extremes Property of Proportions	In a proportion, the product of the extremes is equal to the product of the means. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example: Solve $\frac{x}{5} = \frac{10}{13}$.

$$\frac{x}{5} = \frac{10}{13}$$
$$13x = 50$$
$$x = 3\frac{11}{13}$$

The solution is $3\frac{11}{13}$.

Solve each proportion.

1. $\frac{0.1}{2} = \frac{0.5}{x}$
2. $\frac{x+1}{4} = \frac{3}{4}$
3. $\frac{4}{6} = \frac{8}{x}$
4. $\frac{x}{21} = \frac{3}{63}$
5. $\frac{9}{y+1} = \frac{18}{54}$
6. $\frac{3-x}{4+x} = \frac{8}{48}$
7. $\frac{4x}{25} = \frac{85-x}{100}$
8. $\frac{x+8}{-3} = \frac{17-x}{-2}$

Use a proportion to solve each problem.

9. To make a model of the Guadelupe River bed, Hermie used 1 inch of clay for 5 miles of the actual river's length. His model river was 50 inches long. How long is the Guadelupe River?

10. Josh finished 24 math problems in one hour. At that rate, how many hours will it take him to complete 72 problems?

NAME ________________________ DATE ____________

Study Guide

Student Edition
Pages 215–221

Percents

A percent problem may be easier to solve if a proportion is used.

Percent Proportion
$\frac{\text{percentage}}{\text{base}} = \text{rate}$ or $\frac{\text{percentage}}{\text{base}} = \frac{r}{100}$

Example 1: 25 is what percent of 30?

$$\frac{\text{percentage} \rightarrow 25}{\text{base} \rightarrow 30} = \frac{r}{100} \leftarrow \text{rate}$$

$$2500 = 30r$$

$$83\frac{1}{3} = r$$

25 is $83\frac{1}{3}$% of 30.

Example 2: What number is 24% of 200?

$$\frac{\text{percentage} \rightarrow n}{\text{base} \rightarrow 200} = \frac{24}{100} \leftarrow \text{rate}$$

$$n = \frac{24}{100}(200)$$

$$= 48$$

48 is 24% of 200.

Use a proportion to answer each question.

1. Eight is what percent of 20?

2. Thirty is what percent of 50?

3. What is 75% of 24?

4. Find 60% of 90.

5. Twelve is 20% of what number?

6. 19.3 is 25% of what number?

7. On Wednesday Jean's Nursery received a shipment of 60 flowering crabapple trees. Jean had ordered 80 trees. What percent of her order arrived on Wednesday?

8. Phil received a commission of 5% on the sale of a house. If the amount of his commission was $4780, what was the selling price of the house?

NAME ______________________ DATE ________

Study Guide

Student Edition
Pages 222–227

Percent of Change

Some percent problems involve finding a percent of increase or decrease.

Percent of Increase	Percent of Decrease
A coat that cost \$50 last year costs \$55 this year. The price increased by \$5 since last year. amount of increase → / original price → $\frac{5}{50} = \frac{r}{100}$ $500 = 50r$ $10 = r$, or $r = 10$ The percent of increase is 10%.	Slacks that originally cost \$30 are now on sale for \$22. Find the percent of decrease. amount of decrease → / original price → $\frac{8}{30} = \frac{r}{100}$ $800 = 30r$ $26\frac{2}{3} = r$, or $r = 26\frac{2}{3}$ The percent of decrease is $26\frac{2}{3}$%, or about 27%.

The sales tax on a purchase is a percent of the purchase price. To find the total price, you must calculate the amount of sales tax and add it to the purchase price.

Find the final price of each item. When there is a discount and sales tax, compute the discount price first.

1. Compact Disc: \$16.00
Discount: 15%

2. Two concert tickets: \$28.00
Student discount: 28%

3. Airline Ticket: \$248.00
Superair discount: 33%

4. Celebrity Photo Calendar: \$10.95
Sales tax: 7.5%

5. Class Ring: \$89.00
Group discount: 17%
Sales tax: 5%

6. Computer Software: \$44.00
Discount: 21%
Sales tax: 6%

Solve each problem.

7. The original selling price of a new sports video was \$65.00. Due to demand the price was increased to \$87.75. What was the percent of increase over the original price?

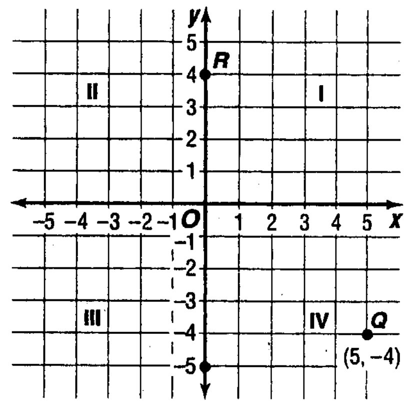
8. A high school paper increased its sales by 75% when it ran an issue featuring a contest to win a class party. Before the contest issue, 10% of the school's 800 students bought the paper. How many students bought the contest issue?

NAME ______________________ DATE ____________

Study Guide

Student Edition
Pages 254–259

The Coordinate Plane



In the diagram at the right, the two perpendicular lines, called the x-axis and the y-axis, divide the coordinate plane into Quadrants I, II, III, and IV. The point where the two axes intersect is called the origin. The origin is represented by the ordered pair (0, 0).

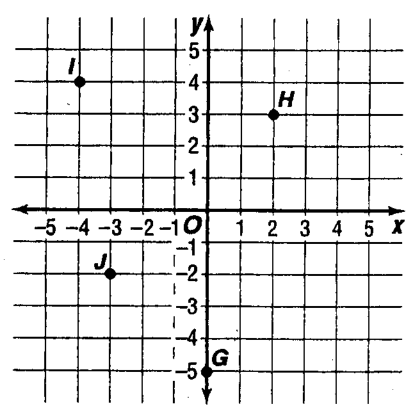
Every other point in the coordinate plane is also represented by an ordered pair of numbers. The ordered pair for point Q is $(5, -4)$. We say that 5 is the x-coordinate of Q and -4 is the y-coordinate of Q.

Example: Write the ordered pair for the point R above.

The x-coordinate is 0 and the y-coordinate is 4. Thus, the ordered pair for R is (0, 4).

To graph any ordered pair (x, y), begin at the origin. Move left or right x units. From there, move up or down y units. Draw a dot at that point.

Graph each point on the coordinate plane at the right.



1. A (0, 0) **2.** B (5, 0)

3. C $(-3, 4)$ **4.** D $(4, -5)$

5. E $(-2, -3)$ **6.** F $(2, -1)$

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located.

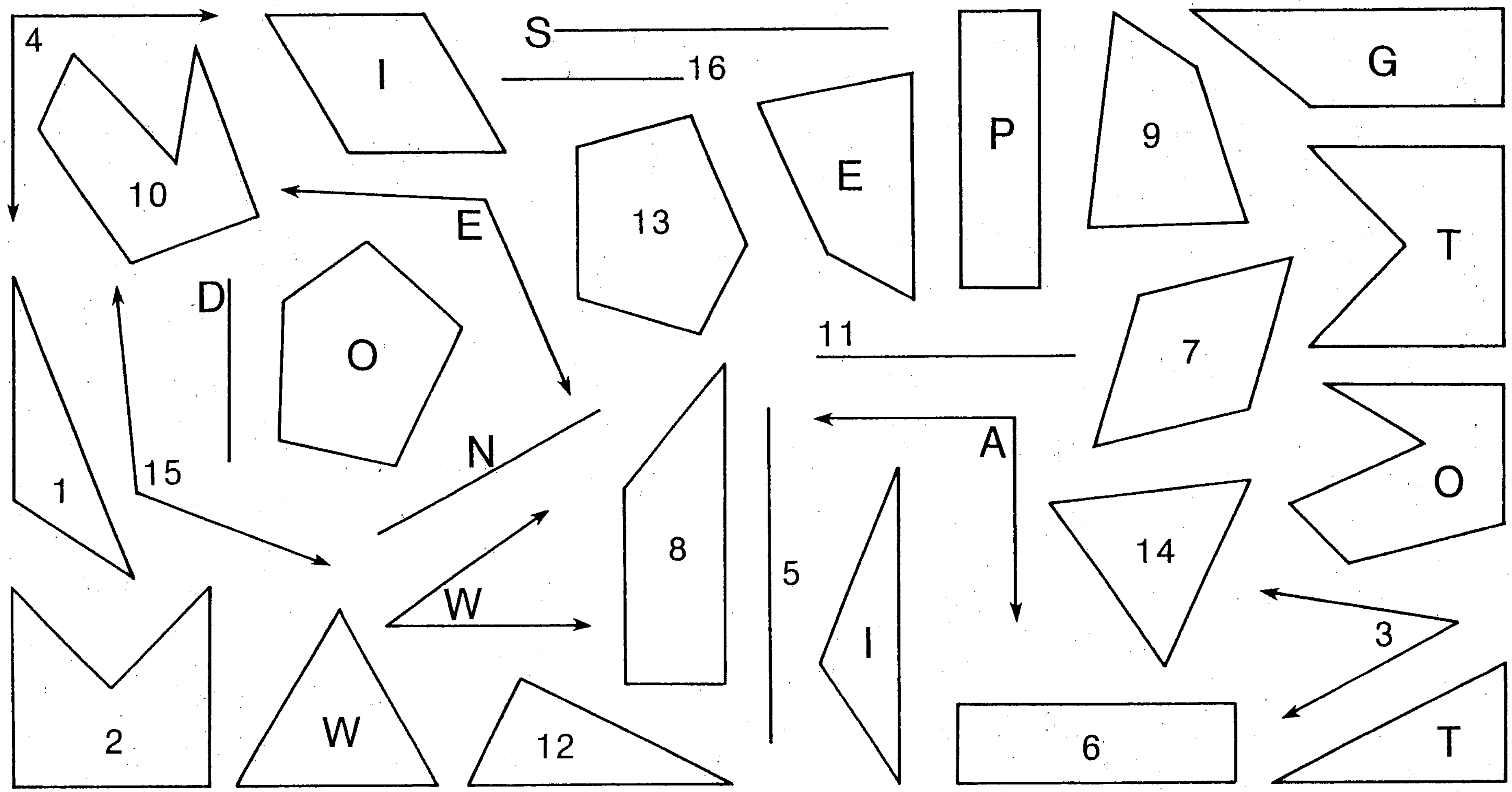
7. G **8.** H **9.** I **10.** J

How Did The Baby Pigeon Manage To Fly South In The Winter?

TO ANSWER THE TITLE QUESTION:

Find a pair of CONGRUENT FIGURES below. One of them will have a number and the other will have a letter. The number tells you where to put the letter in the boxes at the bottom of the page.

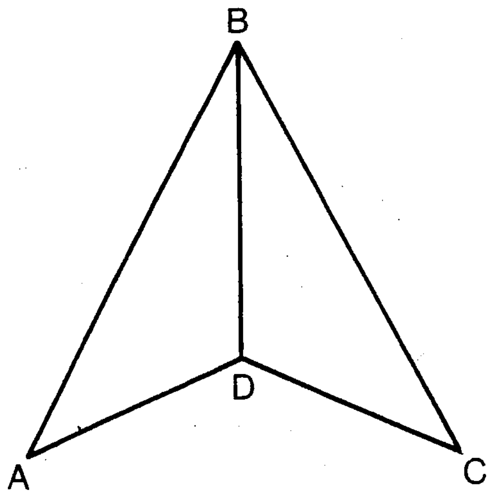
KEEP WORKING AND YOU WILL DISCOVER THE PUNNY ANSWER.



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

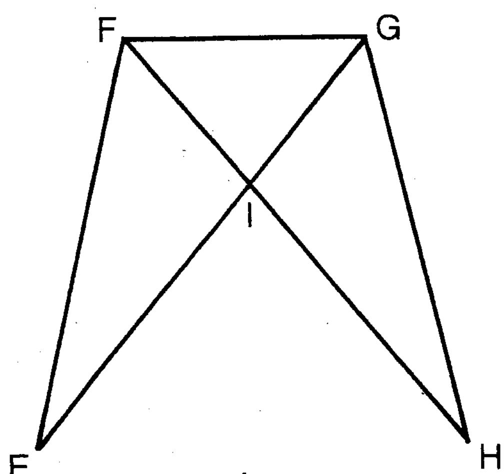
Triangle Treat

All the sides and angles are listed for each triangle. Find a pair of CORRESPONDING PARTS. One will have a number, and the other will have a letter. Write the letter in the box at the bottom of the page that contains the number of the corresponding part.



$\triangle ABD \cong \triangle CBD$

(1) $\angle A$	(S) $\overline{BD}$
(2) $\angle ABD$	(I) $\angle CDB$
(3) $\angle BDA$	(E) $\angle C$
(4) $\overline{AB}$	(A) $\overline{CD}$
(5) $\overline{BD}$	(E) $\overline{BC}$
(6) $\overline{AD}$	(O) $\angle DBC$

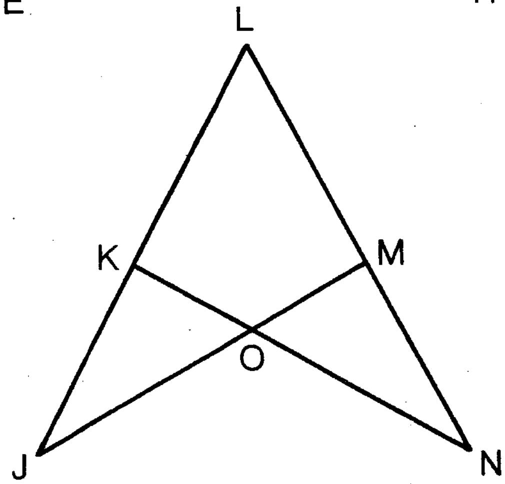


$\triangle EFG \cong \triangle HGF$

(7) $\angle E$	(R) $\overline{FH}$
(8) $\angle EFG$	(O) $\angle H$
(9) $\angle FGE$	(E) $\angle FGH$
(10) $\overline{EF}$	(E) $\overline{FG}$
(11) $\overline{FG}$	(I) $\angle GFH$
(12) $\overline{GE}$	(S) $\overline{GH}$

$\triangle EFI \cong \triangle HGI$

(13) $\angle E$	(S) $\angle HGI$
(14) $\angle EFI$	(C) $\overline{IH}$
(15) $\angle FIE$	(U) $\angle GIH$
(16) $\overline{FE}$	(E) $\overline{GH}$
(17) $\overline{FI}$	(E) $\angle H$
(18) $\overline{IE}$	(R) $\overline{GI}$



$\triangle JLM \cong \triangle NLK$

(19) $\angle J$	(T) $\angle LKN$
(20) $\angle L$	(R) $\angle L$
(21) $\angle LMJ$	(T) $\overline{KN}$
(22) $\overline{LJ}$	(V) $\overline{LN}$
(23) $\overline{LM}$	(D) $\overline{LK}$
(24) $\overline{MJ}$	(E) $\angle N$

$\triangle JKO \cong \triangle NMO$

(25) $\angle J$	(C) $\overline{ON}$
(26) $\angle JKO$	(P) $\overline{MO}$
(27) $\angle KOJ$	(S) $\angle NMO$
(28) $\overline{KJ}$	(N) $\angle MON$
(29) $\overline{KO}$	(T) $\angle N$
(30) $\overline{OJ}$	(T) $\overline{MN}$

5	25	7	17	8	23	13	28	1	18	24	3	22	11	14	6	20	16	30	2	15	27	21	4	12	26	29	9	19	10

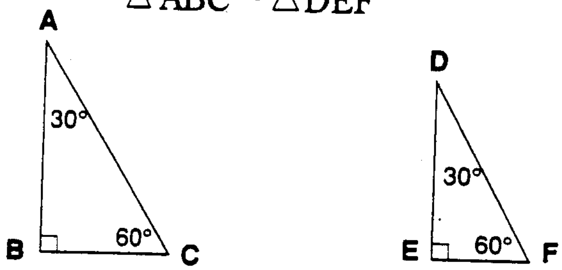
Corresponding Parts in Similar Figures

Matching parts in similar figures are also called corresponding parts. Enlarging or reducing a figure changes the lengths of the sides, but the sizes of the angles remain the same.

Example

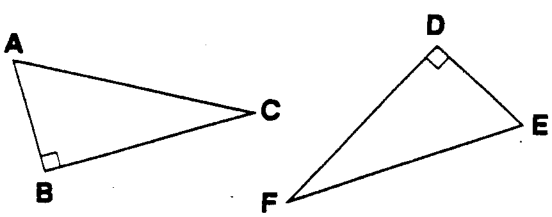
symbol for similar

$\triangle ABC \sim \triangle DEF$



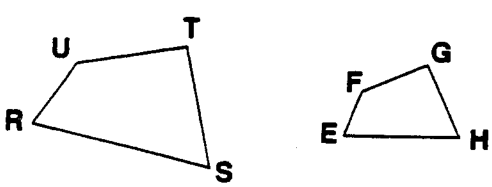
Corresponding Angles	Corresponding Sides
$\angle A = \angle D$	$\overline{AB} \sim \overline{DE}$
$\angle B = \angle E$	$\overline{BC} \sim \overline{EF}$
$\angle C = \angle F$	$\overline{AC} \sim \overline{DF}$

▶ Write the corresponding angles and sides for the similar figures.



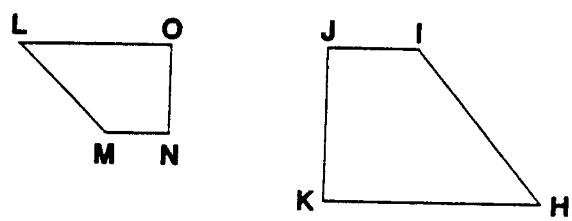
Corresponding Angles / Corresponding Sides

1. a) $\angle A = \angle$ _____ d) $\overline{AB}$ and _____
   b) $\angle B = \angle$ _____ e) $\overline{AC}$ and _____
   c) $\angle C = \angle$ _____ f) $\overline{BC}$ and _____



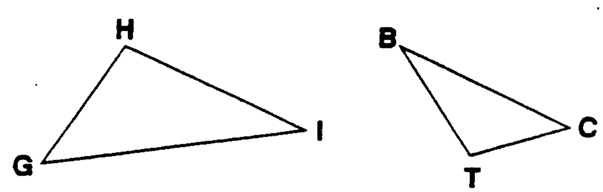
Corresponding Angles / Corresponding Sides

2. a) $\angle R = \angle$ _____ e) $\overline{RU}$ and _____
   b) $\angle U = \angle$ _____ f) $\overline{ST}$ and _____
   c) $\angle T = \angle$ _____ g) $\overline{RS}$ and _____
   d) $\angle S = \angle$ _____ h) $\overline{UT}$ and _____



Corresponding Angles / Corresponding Sides

3. a) $\angle O = \angle$ _____ e) $\overline{NO}$ and _____
   b) $\angle M = \angle$ _____ f) $\overline{LM}$ and _____
   c) $\angle L = \angle$ _____ g) $\overline{OL}$ and _____
   d) $\angle N = \angle$ _____ h) $\overline{MN}$ and _____



Corresponding Angles / Corresponding Sides

4. a) $\angle G = \angle$ _____ d) $\overline{GH}$ and _____
   b) $\angle I = \angle$ _____ e) $\overline{GI}$ and _____
   c) $\angle H = \angle$ _____ f) $\overline{HI}$ and _____

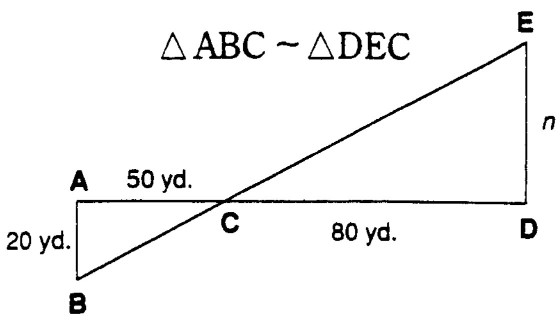
Similar Triangles to Measure

Many problems can be solved using similar triangles. You can use a proportion to help you find the length of a missing side in one of two similar triangles.

Example: Find the length of $\overline{DE}$.

Set up and solve a proportion using the lengths of the corresponding sides. Let $\overline{DE} = n$.

$\triangle ABC \sim \triangle DEC$



smaller triangle / larger triangle $\frac{50}{80} = \frac{20}{n}$ smaller triangle / larger triangle

$$50 \times n = 20 \times 80$$

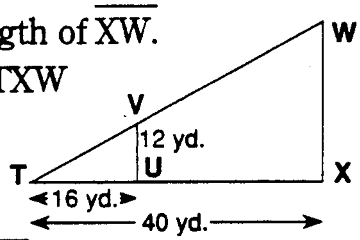
$$\frac{50 \times n}{50} = \frac{1{,}600}{50}$$

$$n = 32$$

The length of $\overline{DE}$ is 32 yards.

▶ Set up a proportion to find the unknown length. Let n represent the unknown length.

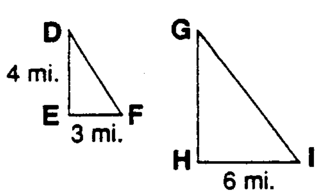
1. Find the length of $\overline{XW}$.
$\triangle TUV \sim \triangle TXW$



$\frac{\square}{\square} = \frac{\square}{\square}$

$n =$ ________

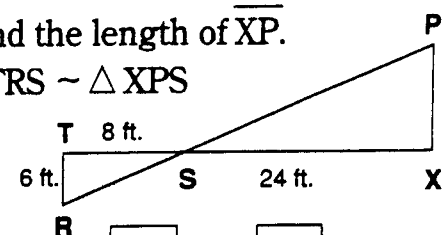
2. Find the length of $\overline{GH}$.
$\triangle DEF \sim \triangle GHI$



$\frac{\square}{\square} = \frac{\square}{\square}$

$n =$ ________

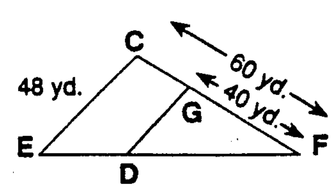
3. Find the length of $\overline{XP}$.
$\triangle TRS \sim \triangle XPS$



$\frac{\square}{\square} = \frac{\square}{\square}$

$n =$ ________

4. Find the length of $\overline{GD}$.
$\triangle FCE \sim \triangle FGD$



$\frac{\square}{\square} = \frac{\square}{\square}$

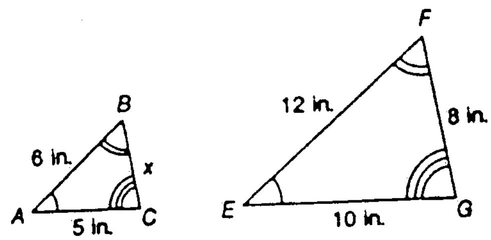
$n =$ ________

Name ____________________ Date ____________________

Practice Worksheet 12-9

Similar Triangles

Use the similar triangles at the right to answer Exercises 1–3.



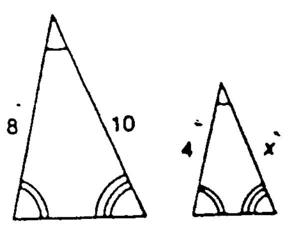
1. List three proportions for $\triangle ABC$ and $\triangle EFG$.

2. What side corresponds to $\overline{AB}$

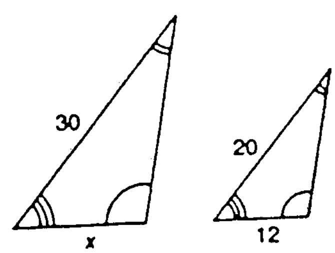
3. Find the value of x.

Find the value of x in each pair of similar triangles.

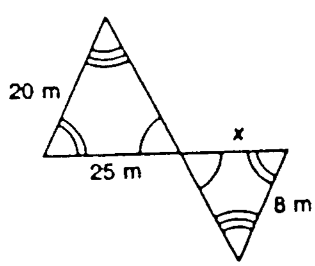
4.



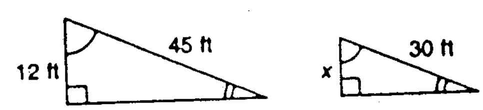
5.



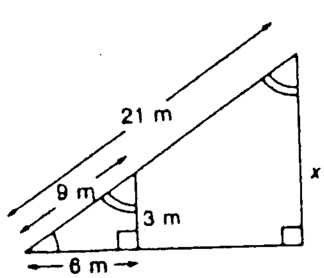
6.



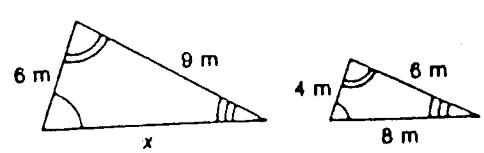
7.



8.



9.

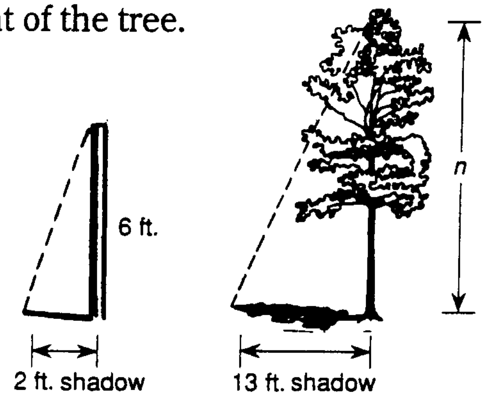


Similar Triangles and Indirect Measurement

Using proportions can be a great help in finding distances you cannot measure directly.

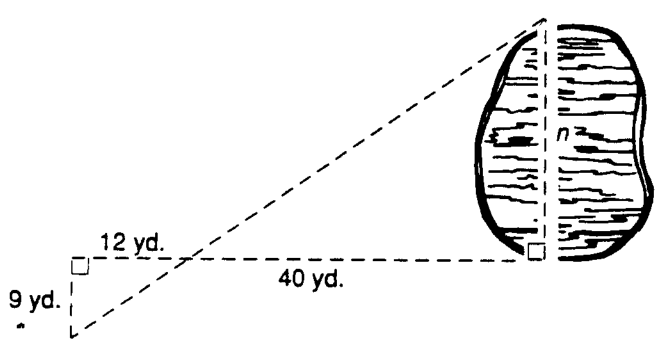
▶ Use a proportion to solve the problems below.

1. Using a pole and shadows, find the height of the tree.



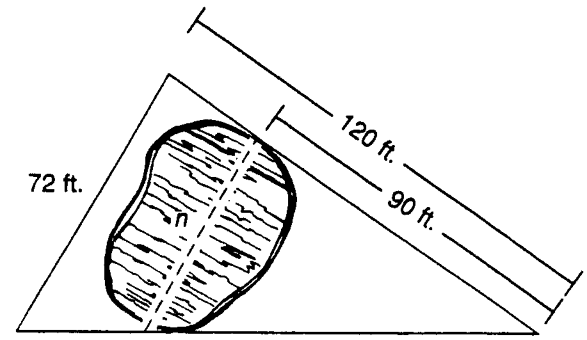
$n =$ ______

2. What is the distance across the lake?



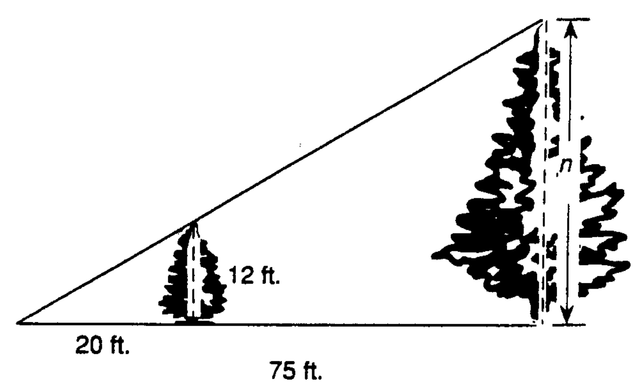
$n =$ ______

3. Find the distance across the pond.



$n =$ ______

4. Use the short tree to find the height of the tall tree.



$n =$ ______

✕✕ DOUBLE CROSS ✕✕

1. WHAT DO YOU GET WHEN YOU CROSS A VAMPIRE WITH A TURTLE?

____

$\frac{-7}{18}$ $\frac{-39}{40}$ $\frac{-3}{10}$ $-1\frac{1}{15}$ $-1\frac{1}{3}$ $\frac{9}{20}$ $\frac{1}{3}$ $\frac{-13}{15}$ $\frac{67}{100}$ $\frac{-39}{40}$ $\frac{-7}{18}$ $-1\frac{13}{24}$ $\frac{1}{18}$ $\frac{5}{12}$ $-1\frac{1}{15}$ $\frac{-3}{10}$

2. WHAT DO YOU GET WHEN YOU CROSS A MOTORCYCLE WITH A JOKE BOOK?

____

$\frac{-7}{18}$ $-1\frac{1}{3}$ $\frac{-7}{18}$ $-1\frac{13}{24}$ $\frac{-7}{18}$ $\frac{17}{24}$ $\frac{-7}{18}$ $\frac{17}{24}$ $\frac{-7}{18}$ $\frac{17}{24}$ $\frac{-7}{18}$ $\frac{17}{24}$ $\frac{-7}{18}$

3. WHAT DO YOU GET WHEN YOU CROSS FIVE PIGS WITH FIVE DEER?

____

$\frac{19}{36}$ $\frac{-3}{10}$ $\frac{29}{48}$ $\frac{9}{20}$ $\frac{-13}{15}$ $\frac{67}{100}$ $\frac{9}{20}$ $\frac{-7}{18}$ $\frac{29}{48}$ $\frac{1}{12}$ $\frac{-17}{30}$ $\frac{-1}{20}$ $\frac{1}{4}$ $-1\frac{5}{24}$ $\frac{9}{20}$

TO DECODE THE ANSWERS TO THESE THREE QUESTIONS:
Do any exercise below and find your answer in the code. Each time the answer appears in the code, write the letter of that exercise above it.
KEEP WORKING AND YOU WILL DISCOVER WHAT YOU GET FROM EACH DOUBLE CROSS!

I $\frac{2}{3} + \frac{-1}{4} =$

E $\frac{-4}{5} + \frac{1}{2} =$

K $\frac{-1}{3} + \frac{-7}{8} =$

U $\frac{-4}{5} + \frac{3}{4} =$

O $\frac{-1}{5} + \frac{-2}{3} =$

C $\frac{5}{6} + \frac{-7}{12} =$

D $\frac{-3}{4} + \frac{5}{6} =$

R $\frac{-9}{10} + \frac{-1}{6} =$

T $\frac{-1}{4} + \frac{7}{9} =$

L $\frac{11}{15} + \frac{-2}{5} =$

M $\frac{-11}{12} + \frac{-5}{8} =$

N $\frac{2}{3} + \frac{-1}{16} =$

P $\frac{-4}{9} + \frac{1}{2} =$

Y $\frac{-3}{4} + \frac{-7}{12} =$

V $\frac{-3}{5} + \frac{-3}{8} =$

W $\frac{3}{10} + \frac{37}{100} =$

B $\frac{3}{10} + \frac{-13}{15} =$

H $\frac{-1}{8} + \frac{5}{6} =$

A $\frac{-1}{6} + \frac{-2}{9} =$

S $\frac{-1}{4} + \frac{7}{10} =$

SUM CODE

Do any exercise below and find your answer in the answer columns. Notice the number in front of the answer. Each time this number appears in the code, write the letter of the exercise above it. Keep working and you will decode the message.

(S) $-1\frac{1}{4} + {}^{-}2\frac{1}{2} =$

(O) $-3\frac{2}{3} + {}^{-}1\frac{2}{5} =$

(A) $4\frac{1}{2} + {}^{-}2\frac{1}{3} =$

(F) $3\frac{1}{6} + {}^{-}5\frac{3}{5} =$

(U) $-8\frac{3}{4} + 1\frac{3}{10} =$

(T) $-7\frac{1}{3} + 7\frac{3}{4} =$

(M) $-2\frac{1}{16} + {}^{-}2\frac{1}{3} =$

(L) $6\frac{3}{7} + {}^{-}4\frac{1}{4} =$

(D) $-1\frac{1}{6} + 5\frac{7}{10} =$

(N) $4\frac{2}{9} + {}^{-}9\frac{1}{2} =$

(W) $-8\frac{3}{4} + 1\frac{2}{5} =$

(C) $-3\frac{1}{4} + {}^{-}5\frac{7}{9} =$

(G) $6\frac{8}{11} + 2\frac{2}{3} =$

(I) $5\frac{5}{6} + {}^{-}5\frac{8}{9} =$

(H) $-3\frac{4}{5} + 2\frac{3}{10} =$

(R) $8\frac{3}{8} + {}^{-}9\frac{2}{3} =$

(E) $-4\frac{1}{5} + {}^{-}1\frac{7}{8} =$

(B) $-7\frac{3}{8} + 7\frac{3}{8} =$

ANSWERS

1	$-1\frac{7}{24}$	10	$\frac{-1}{18}$
2	$-5\frac{1}{15}$	11	$\frac{5}{12}$
3	$-7\frac{9}{20}$	12	$4\frac{8}{15}$
4	$-6\frac{3}{40}$	13	$-1\frac{1}{2}$
5	$-5\frac{5}{18}$	14	$-3\frac{3}{4}$
6	$9\frac{13}{33}$	15	$-4\frac{19}{48}$
7	0	16	$-7\frac{7}{20}$
8	$-2\frac{13}{30}$	17	$2\frac{1}{6}$
9	$2\frac{5}{28}$	18	$-9\frac{1}{36}$

17·7·4·17·1·15·3·14·11·7·4·16·17·1·4·16·13·4·5·9·2·14·10·5·6

13·10·14·13·17·10·1·7·4·18·17·3·14·4·17·8·11·4·1 · 17·9·9· 13·2·16

15·3·18·13·18·2·9·12·17·10·1·18·17·5·17·7·17·1·4·7·4·17·1·7·4·17·1?